BVSR ≠ Buffy Vampire Slayer Relationships
Creative Problem Solving as Variation-Selection:
The Blind-Sighted Continuum and Solution Variant Typology
Background

• Donald T. Campbell’s (1960) BVSR model of creativity and discovery
• Controversies and confusions
• Need for a formal
  – variant typology
  – blind-sighted metric
• expressed in terms of creative problem solving (to keep discussion simple)
Definitions

• Given problem:
  – Goal with attainment criteria
  – For complex problems: subgoals with their separate attainment criteria
  – Goals and subgoals may form a goal hierarchy

  • e.g., writing a poem: the composition’s topic or argument, its length and structure, meter or rhythm, rhyme and alliteration, metaphors and similes, and the best word for a single place that optimizes both sound and sense (cf. Edgar Allan Poe’s 1846 “The Philosophy of Composition”)
Definitions

• Solution variants:
  – two or more alternative solutions or parts of solutions
  – algorithms, analogies, arrangements, assumptions, axioms, colors, conjectures, corollaries, definitions, designs, equations, estimates, explanations, expressions, forms, formulas, harmonies, heuristics, hypotheses, images, interpretations, media, melodies, metaphors, methods, models, narratives, observations, parameters, patterns, phrasings, plans, predictions, representations, rhymes, rhythms, sketches, specifications, start values, statistics, structures, techniques, terms, themes, theorems, theories, words, etc.
  – depending on nature of problem
Definitions

• Creative solution (Boden, 2004; USPTO):
  – novel (or original)
  – useful (or functional, adaptive, or valuable)
  – surprising (or “nonobvious”)
    • innovations, not adaptations
    • inventions, not improvements
    • productive, not reproductive thought
Definitions

• Variant parameters: $X$ characterized by:
  - generation probability: $p$
  - solution utility: $u$ (probability or proportion)
    • probability of selection-retention
    • proportion of $m$ criteria actually satisfied
  - selection expectation: $v$ (i.e., the individual’s implicit or explicit knowledge of the utility and therefore likely selection and retention)
### $k$ Hypothetical Solution Variants

<table>
<thead>
<tr>
<th>Solution</th>
<th>Probability</th>
<th>Utility</th>
<th>Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>$p_1$</td>
<td>$u_1$</td>
<td>$v_1$</td>
</tr>
<tr>
<td>$X_2$</td>
<td>$p_2$</td>
<td>$u_2$</td>
<td>$v_2$</td>
</tr>
<tr>
<td>$X_3$</td>
<td>$p_3$</td>
<td>$u_3$</td>
<td>$v_3$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$X_i$</td>
<td>$p_i$</td>
<td>$u_i$</td>
<td>$v_i$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$X_k$</td>
<td>$p_k$</td>
<td>$u_k$</td>
<td>$v_k$</td>
</tr>
</tbody>
</table>

$$0 \leq p_i \leq 1, 0 \leq u_i \leq 1, 0 \leq v_i \leq 1$$
## Solution Variant Typology

<table>
<thead>
<tr>
<th>Type</th>
<th>$p_i$</th>
<th>$u_i$</th>
<th>$v_i$</th>
<th>Generation</th>
<th>Prospects</th>
<th>Prior knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>likely</td>
<td>true positive</td>
<td>utility known</td>
</tr>
<tr>
<td>2</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>likely</td>
<td>true positive</td>
<td>utility unknown</td>
</tr>
<tr>
<td>3</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$= 0$</td>
<td>likely</td>
<td>false positive</td>
<td>utility unknown</td>
</tr>
<tr>
<td>4</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>likely</td>
<td>false positive</td>
<td>utility known(^1)</td>
</tr>
<tr>
<td>5</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
<td>unlikely</td>
<td>false negative</td>
<td>utility known</td>
</tr>
<tr>
<td>6</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>unlikely</td>
<td>false negative</td>
<td>utility unknown</td>
</tr>
<tr>
<td>7</td>
<td>$= 0$</td>
<td>$= 0$</td>
<td>$= 0$</td>
<td>unlikely</td>
<td>true negative</td>
<td>utility unknown</td>
</tr>
<tr>
<td>8</td>
<td>$= 0$</td>
<td>$= 0$</td>
<td>$&gt; 0$</td>
<td>unlikely</td>
<td>true negative</td>
<td>utility known(^2)</td>
</tr>
</tbody>
</table>

\(^1\)To avoid confirmation bias \(^2\)Often resulting from prior BVSR trials
Two Special Types

• Reproductive Type 1:
  – $p_i = u_i = v_i = 1$
  – i.e., low novelty, high utility, low surprise
  – BVSR unnecessary because variant “frontloaded” by known utility value
  – Selection becomes mere “quality control” to avoid calculation mistakes or memory slips
  – But also routine, even algorithmic thinking, and hence not creative
Two Special Types

• Creative Type 2:
  – $p_i \neq 0$ but $p_i \approx 0$ (high novelty)
  – $u_i = 1$ (high utility)
  – $v_i = 0$ or $v_i \approx 0$ (high surprise)
  – BVSR mandatory to distinguish from Type 3
  – Because the creator *does not know* the utility value, must generate and test
  – Hence, innovative, inventive, productive, or creative thinking
Quantitative Creativity Measure

- \( c_i = (1 - p_i)u_i(1 - v_i) \)
- where \( 0 \leq c_i < 1 \)
- \( c_i \rightarrow 1 \) as
  - \( p_i \rightarrow 0 \) (maximizing novelty),
  - \( u_i \rightarrow 1 \) (maximizing utility), and
  - \( v_i \rightarrow 0 \) (maximizing surprise)
- \( c_i = 0 \) when \( p_i = 1 \) and \( v_i = 1 \) regardless of \( u_i \)
- perfectly productive variant \( p_i = u_i = v_i = 1 \)
Quantitative Creativity Measure

- Less extreme examples:
  - \( p_i = 0.100, u_i = 1.000, v_i = 0.100, c_i = 0.810 \)
  - \( p_i = 0.100, u_i = 0.500, v_i = 0.100, c_i = 0.405 \)

- Individualistic vs. collectivistic cultures:
  - \( p_1 = 0.001 \) and \( u_1 = 0.500 \) (novelty > utility)
  - \( p_2 = 0.500 \) and \( u_2 = 1.000 \) (novelty < utility)
  - letting \( v_1 = v_2 = 0 \)
  - \( c_1 \approx 0.500 \) (or .4995, exactly)
  - \( c_2 = 0.500 \)
Blind-Sighted Continuum

• Goal: a measure for any set of $k$ variants

• Blind-sighted metric: Start with Tucker’s $\varphi$
  
  $\varphi_{pu} = \langle p, u \rangle / \langle p, p \rangle^{1/2} \langle u, u \rangle^{1/2}$, or
  
  $\varphi_{pu} = \sum p_i u_i / (\sum p_i^2 \sum u_i^2)^{1/2}$ over all $k$ variants

• $0 \leq \varphi \leq 1$
  
  – If $.85-.94$, then factors/pcs reasonably alike
  
  – If $\varphi > .95$, then factors/pcs equal (Lorenzo-Seva & ten Berge, 2006)

• But we will use $\varphi^2$, where $0 \leq \varphi^2 \leq 1$
Representative Calculations

• For $k = 2$
  
  - If $p_1 = 1$, $p_2 = 0$, $u_1 = 1$, $u_2 = 0$, $\phi_{pu}^2 = 1$
    • i.e., perfect sightedness ("perfect expertise")
  
  - If $p_1 = 1$, $p_2 = 0$, $u_1 = 0$, $u_2 = 1$, $\phi_{pu}^2 = 0$
    • i.e., perfect blindness ("bad guess")
  
  - If $p_1 = .5$, $p_2 = .5$, $u_1 = 1$, $u_2 = 0$, $\phi_{pu}^2 = .5$
    • midpoint on blind-sighted continuum
    • e.g., fork-in-the-road problem
Representative Calculations

• For $k \geq 2$
  – Equiprobability with only one unity utility
    • $p_i = 1/k$
    • $\varphi_{pu}^2 = (1/k)^2/(1/k) = 1/k$
  – $\varphi_{pu}^2$ yields the average per-variant probability of finding a useful solution in the $k$ variants
  – Therefore …
Representative Calculations

- $k = 2, \quad \varphi_{pu}^2 = .500$ (given earlier);
- $k = 3, \quad \varphi_{pu}^2 = .333$;
- $k = 4, \quad \varphi_{pu}^2 = .250$;
- $k = 5, \quad \varphi_{pu}^2 = .200$;
- $k = 6, \quad \varphi_{pu}^2 = .167$;
- $k = 7, \quad \varphi_{pu}^2 = .143$;
- $k = 8, \quad \varphi_{pu}^2 = .125$;
- $k = 9, \quad \varphi_{pu}^2 = .111$;
- $k = 10, \quad \varphi_{pu}^2 = .100$; etc.
Representative Calculations

• For $k \geq 2$
  – Equiprobability with only one zero utility
    • $k = 4$
    • $p_1 = p_2 = p_3 = p_4 = 0.25$, $u_1 = 0$, $u_2 = u_3 = u_4 = 1$
    • $\phi_{pu}^2 = 0.75$ (i.e., average probability of solution 3/4)
    • N.B.: $\sqrt{0.75} \approx 0.87$ minimum for Tucker’s $\phi$

• Hence, the following partitioning …
Four Sectors

- First: Effectively blind
  - \( .00 \leq \varphi_{pu}^2 \leq .25 = Q_1 \) (1st quartile)

- Second: Mostly blind but partially sighted
  - \( .25 < \varphi_{pu}^2 \leq .50 = Q_2 \) (2nd quartile)

- Third: Mostly sighted but partially blind
  - \( .50 < \varphi_{pu}^2 \leq .75 = Q_3 \) (3rd quartile)

- Fourth: Effectively sighted
  - \( .75 < \varphi_{pu}^2 \leq 1.0 \)
  - “pure” sighted if \( \varphi_{pu}^2 > .90 \approx .95^2 \)
Connection with Typology

- $\varphi_{pu}^2$ tends to increase with more variant Types 1 and 2 ($ps > 0$ and $us > 0$)
- $\varphi_{pu}^2$ always decreases with more variant Types 3 and 4 ($ps > 0$ and $us = 0$)
- $\varphi_{pu}^2$ always decreases with more variant Types 5 and 6 ($ps = 0$ and $us > 0$)
- $\varphi_{pu}^2$ neither increases nor decreases with variant Types 7 and 8 ($ps = 0$ and $us = 0$)
Selection Procedures

• External versus Internal
  – Introduces no complications

• Simultaneous versus Sequential
  – Introduces complications
Sequential Selection

• Need to add an index for consecutive trials to allow for changes in the parameter values:

  • $p_{1t}, p_{2t}, p_{3t}, \ldots p_{it}, \ldots p_{kt}$
  • $u_{1t}, u_{2t}, u_{3t}, \ldots u_{it}, \ldots u_{kt}$
  • $v_{1t}, v_{2t}, v_{3t}, \ldots v_{it}, \ldots v_{kt}$
  • where $t = 1, 2, 3, \ldots n$ (number of trials)
  • Then still, $0 \leq \varphi_{pu}^2(t) \leq 1$, but
  • $\varphi_{pu}^2(t) \rightarrow 1$ as $t \rightarrow n$ (Type 3 to Type 8)
Caveat: Pro-Sightedness Bias

- Because $\phi_{pu}^2$ increases with Type 2 though $\nu_i = 0$, it could reflect chance concurrences between $p$ and $u$
  - e.g., lucky response biases
- Hence, superior measure would use
  - $\phi_{pw}^2 = (\sum p_i w_i) / (\sum p_i^2 \sum w_i^2)$,
  - where $w_i = u_i \nu_i$, and hence $\phi_{pw}^2 < \phi_{pu}^2$
- But $\nu_i$ is seldom known, so …
Concrete Illustrations

- Edison’s “drag hunts”
- Picasso’s horse sketches for *Guernica*
- Kepler’s Third Law
- Watson’s discovery of the DNA base pairs
Edison’s “drag hunts”

- For lamp filaments, battery electrodes, etc.
- Incandescent filament utility criteria:
  - (1) low-cost,
  - (2) high-resistance,
  - (3) brightly glow 13½ hours, and
  - (4) durable
Edison’s “drag hunts”

• Tested hundreds of possibilities:
  – 100 trial filaments: $\phi_{pu}^2 \approx .01$ ($1^{st}$ percentile)
  – 10 trial filaments: $\phi_{pu}^2 \approx .1$ ($1^{st}$ decile)

• These two estimates do not require equiprobability, only $p-u$ “decoupling”

• e.g., same results emerge when both $p$ and $u$ are vectors of random numbers with positively skewed distributions (i.e., the drag hunts are “purely blind”)

Picasso’s *Guernica* Sketches

- 21 horse sketches represent the following solution variants with respect to the head:
  - $X_1 =$ head thrusting up almost vertically: 1, 2, and 3 (top)
  - $X_2 =$ head on the left side, facing down: 4 and 20
  - $X_3 =$ head facing up, to the right: 5, 6, 7, 8, 9, and 11
  - $X_4 =$ head upside down, to right, facing down, turned left: 10, 12, and 13
  - $X_5 =$ head upside down, to left, facing down, turned left: 15
  - $X_6 =$ head upside down, to right, facing down, pointed right: 17
  - $X_7 =$ head level, facing left: 3 (bottom), 18 (top), 18 (bottom), 28, and 29
- Yielding …
Probabilities and Utilities

- $p_1 = 3/21 = .143$
- $p_2 = 2/21 = .095$
- $p_3 = 6/21 = .286$
- $p_4 = 3/21 = .143$
- $p_5 = 1/21 = .048$
- $p_6 = 1/21 = .048$
- $p_7 = 5/21 = .238$

- $u_1 = 0$
- $u_2 = 0$
- $u_3 = 0$
- $u_4 = 0$
- $u_5 = 0$
- $u_6 = 0$
- $u_7 = 1$
Picasso’s *Guernica* Sketches

- Hence, $\phi_{pu}^2 \approx .293$ (2\textsuperscript{nd} sector, lower end)
- If complications are introduced, e.g.,
  - differentiating more horse variants so $k > 7$,
  - assuming that there are separate whole-part utilities,
- then $\phi_{pu}^2 < .293$ (viz. 1\textsuperscript{st} sector)
(Re)discovering Kepler’s $3^{rd}$ Law

Systematic Search

- $D^1/T^1$ \hspace{1cm} $u_1 = 0$
- $D^1/T^2$ \hspace{1cm} $u_2 = 0$
- $D^2/T^1$ \hspace{1cm} $u_3 = 0$
- $D^2/T^2$ \hspace{1cm} $u_4 = 0$
- $D^2/T^3$ \hspace{1cm} $u_5 = 0$
- $D^3/T^2$ \hspace{1cm} $u_6 = 1$
- $D^3/T^3$ \hspace{1cm} $u_7 = 0$

- $\phi_{pu}^2(1) = .143$
- $\phi_{pu}^2(2) = .167$
- $\phi_{pu}^2(3) = .200$
- $\phi_{pu}^2(4) = .250$
- $\phi_{pu}^2(5) = .333$
- $\phi_{pu}^2(6) = .500$
- Not tested
(Re)discovering Kepler’s 3\textsuperscript{rd} Law

BACON’s Heuristic Search

- $D_1^1/T_1$  \hspace{1cm} $u_1 = 0$
- $D_1^1/T_2$  \hspace{1cm} $u_2 = 0$
- $D_2^1/T_1$  \hspace{1cm} $u_3 = 0$
- $D_2^1/T_2$  \hspace{1cm} $u_4 = 0$
- $D_2^2/T_3$  \hspace{1cm} $u_5 = 0$
- $D_3^2/T_2$  \hspace{1cm} $u_6 = 1$
- $D_3^2/T_3$  \hspace{1cm} $u_7 = 0$

- $\phi_{pu}^2(1) = .143$
- $\phi_{pu}^2(2) = .167$
- Not tested
- Not tested
- Not tested

- $\phi_{pu}^2(6) = .500$
- Not tested
Watson’s Discovery of the DNA Base Pairs

• Four bases (nucleotides):
  – two purines: adenine (A) and guanine (G)
  – two pyrimidines: cytocine (C) and thymine (T)

• Four variants:
  – $X_1 = A-A, G-G, C-C, \text{ and } T-T$
  – $X_2 = A-C \text{ and } G-T$
  – $X_3 = A-G \text{ and } C-T$
  – $X_4 = A-T \text{ and } G-C$
Watson’s Discovery of the DNA Base Pairs

• \(u_1 = 0, u_2 = 0, u_3 = 0, \text{ and } u_4 = 1\)
• where only the last explains Chargaff’s ratios (i.e., \(\%A/\%T = 1 \text{ and } \%G/\%C = 1\))
• But according to Watson’s (1968) report:

  • at \(t = 1\), \(p_{11} \gg p_{21} \approx p_{31} \approx p_{41}: \text{ e.g.,} \)
  
  \[-p_{11} = .40, \quad p_{21} = p_{31} = p_{41} = .20, \quad \phi_{pu}^2(1) = .143\]

  \[-p_{11} = .28, \quad p_{21} = p_{31} = p_{41} = .24, \quad \phi_{pu}^2(1) = .229\]
Conclusions

• First, creative solutions entail Type 2 variants with
  – (a) low generation probabilities (high novelty),
  – (b) high utilities (high usefulness), and
  – (c) low selection expectations (high surprise)
Conclusions

• Second, creative Type 2 variants can only be distinguished from noncreative Type 3 variants by implementing BVSR.

• That is, because the creator does not know the utility in advance, Type 2 and Type 3 can only be discriminated via generation and test episodes.
Conclusions

• Third, $\varphi_{pu}^2$ provides a conservative estimate of where solution variant sets fall on the blind-sighted continuum.

• When $\varphi_{pu}^2$ is applied to real problem-solving episodes, $\varphi_{pu}^2 \leq .5$

• Moreover, variant sets seldom attain even this degree of sightedness until BVSR removes one or more Type 3 variants