BVSR ≠ **Buffy Vampire Slayer Relationships**

Creative Problem Solving as Variation-Selection:

The Blind-Sighted Continuum and Solution Variant Typology

Background

- Donald T. Campbell's (1960) BVSR model of creativity and discovery
- Controversies and confusions
- Need for a formal
  - variant typology
  - blind-sighted metric
- expressed in terms of creative problem solving (to keep discussion simple)

Definitions

- Given problem:
  - Goal with attainment criteria
  - For complex problems: subgoals with their separate attainment criteria
  - Goals and subgoals may form a goal hierarchy
    - e.g., writing a poem: the composition's topic or argument, its length and structure, meter or rhythm, rhyme and alliteration, metaphors and similes, and the best word for a single place that optimizes both sound and sense (cf. Edgar Allan Poe's 1846 "The Philosophy of Composition")

Definitions

- Solution variants:
  - two or more alternative solutions or parts of solutions
  - algorithms, analogies, arrangements, assumptions, axioms, colors, conjectures, corollaries, definitions, designs, equations, estimates, explanations, expressions, forms, formulas, harmonies, heuristics, hypotheses, images, interpretations, media, melodies, metaphors, methods, models, narratives, observations, parameters, patterns, phrasings, plans, predictions, representations, rhymes, rhythms, sketches, specifications, start values, statistics, structures, techniques, terms, themes, theorems, theories, words, etc.
  - depending on nature of problem

Definitions

- Creative solution (Boden, 2004; USPTO):
  - novel (or original)
  - useful (or functional, adaptive, or valuable)
  - surprising (or “nonobvious”)
    - innovations, not adaptations
    - inventions, not improvements
    - productive, not reproductive thought

Definitions

- Variant parameters: *X* characterized by:
  - generation probability: *p*
  - solution utility: *u* (probability or proportion)
    - probability of selection-retention
    - proportion of *m* criteria actually satisfied
  - selection expectation: *v* (i.e., the individual's implicit or explicit knowledge of the utility and therefore likely selection and retention)

k Hypothetical Solution Variants

Solution	Probability	Utility	Expectation
X_1	p_1	u_1	v_1
X_2	p_2	u_2	v_2
X_3	p_3	u_3	v_3
…	…	…	…
X_i	p_i	u_i	v_i
…	…	…	…
X_k	p_k	u_k	v_k

$$0 \leq p_i \leq 1,\ 0 \leq u_i \leq 1,\ 0 \leq v_i \leq 1$$

Solution Variant Typology

Type	p_i	u_i	v_i	Generation	Prospects	Prior knowledge
1	> 0	> 0	> 0	likely	true positive	utility known
2	> 0	> 0	= 0	likely	true positive	utility unknown
3	> 0	= 0	= 0	likely	false positive	utility unknown
4	> 0	= 0	> 0	likely	false positive	utility known[1]
5	= 0	> 0	> 0	unlikely	false negative	utility known
6	= 0	> 0	= 0	unlikely	false negative	utility unknown
7	= 0	= 0	= 0	unlikely	true negative	utility unknown
8	= 0	= 0	> 0	unlikely	true negative	utility known[2]

[1]To avoid confirmation bias [2]Often resulting from prior BVSR trials

Two Special Types

- **Reproductive Type 1:**
  - $p_i = u_i = v_i = 1$
  - i.e., low novelty, high utility, low surprise
  - BVSR unnecessary because variant “frontloaded” by *known* utility value
  - Selection becomes mere “quality control” to avoid calculation mistakes or memory slips
  - But also routine, even algorithmic thinking, and hence not creative

Two Special Types

- **Creative Type 2:**
  - $p_i \neq 0$ but $p_i \approx 0$ (high novelty)
  - $u_i = 1$ (high utility)
  - $v_i = 0$ or $v_i \approx 0$ (high surprise)
  - BVSR mandatory to distinguish from Type 3
  - Because the creator *does not know* the utility value, must generate and test
  - Hence, innovative, inventive, productive, or creative thinking

Quantitative Creativity Measure

- $c_i = (1 - p_i)u_i(1 - v_i)$
- where $0 \leq c_i < 1$
- $c_i \rightarrow 1$ as
  - $p_i \rightarrow 0$ (maximizing novelty),
  - $u_i \rightarrow 1$ (maximizing utility), and
  - $v_i \rightarrow 0$ (maximizing surprise)
- $c_i = 0$ when $p_i = 1$ and $v_i = 1$ regardless of u_i
- perfectly productive variant $p_i = u_i = v_i = 1$

Quantitative Creativity Measure

- Less extreme examples:
  - $p_i = 0.100$, $u_i = 1.000$, $v_i = 0.100$, $c_i = 0.810$
  - $p_i = 0.100$, $u_i = 0.500$, $v_i = 0.100$, $c_i = 0.405$
- Individualistic vs. collectivistic cultures:
  - $p_1 = 0.001$ and $u_1 = 0.500$ (novelty > utility)
  - $p_2 = 0.500$ and $u_2 = 1.000$ (novelty < utility)
  - letting $v_1 = v_2 = 0$
  - $c_1 \approx 0.500$ (or .4995, exactly)
  - $c_2 = 0.500$

Blind-Sighted Continuum

- Goal: a measure for any set of *k* variants
- Blind-sighted metric: Start with Tucker's φ
  - $\varphi_{pu} = \langle \mathbf{p}, \mathbf{u} \rangle / \langle \mathbf{p}, \mathbf{p} \rangle^{1/2} \langle \mathbf{u}, \mathbf{u} \rangle^{1/2}$, or
  - $\varphi_{pu} = \sum p_i u_i / (\sum p_i^2 \sum u_i^2)^{1/2}$ over all *k* variants
- $0 \leq \varphi \leq 1$
  - If .85-.94, then factors/pcs reasonably alike
  - If $\varphi > .95$, then factors/pcs equal (Lorenzo-Seva & ten Berge, 2006)
- But we will use φ^2, where $0 \leq \varphi^2 \leq 1$

Representative Calculations

- For $k = 2$
  - If $p_1 = 1$, $p_2 = 0$, $u_1 = 1$, $u_2 = 0$, $\varphi_{pu}{}^2 = 1$
    - i.e., perfect sightedness ("perfect expertise")
  - If $p_1 = 1$, $p_2 = 0$, $u_1 = 0$, $u_2 = 1$, $\varphi_{pu}{}^2 = 0$
    - i.e., perfect blindness ("bad guess")
  - If $p_1 = .5$, $p_2 = .5$, $u_1 = 1$, $u_2 = 0$, $\varphi_{pu}{}^2 = .5$
    - midpoint on blind-sighted continuum
    - e.g., fork-in-the-road problem

Representative Calculations

- For $k \geq 2$
  - Equiprobability with only one unity utility
    - $p_i = 1/k$
    - $\varphi_{pu}^2 = (1/k)^2/(1/k) = 1/k$
  - φ_{pu}^2 yields the average per-variant probability of finding a useful solution in the k variants
  - Therefore …

Representative Calculations

- $k = 2$, $\varphi_{pu}^2 = .500$ (given earlier);
- $k = 3$, $\varphi_{pu}^2 = .333$;
- $k = 4$, $\varphi_{pu}^2 = .250$;
- $k = 5$, $\varphi_{pu}^2 = .200$;
- $k = 6$, $\varphi_{pu}^2 = .167$;
- $k = 7$, $\varphi_{pu}^2 = .143$;
- $k = 8$, $\varphi_{pu}^2 = .125$;
- $k = 9$, $\varphi_{pu}^2 = .111$;
- $k = 10$, $\varphi_{pu}^2 = .100$; etc.

Representative Calculations

- For $k \geq 2$
  - Equiprobability with only one zero utility
    - $k = 4$
    - $p_1 = p_2 = p_3 = p_4 = .25$, $u_1 = 0$, $u_2 = u_3 = u_4 = 1$
    - $\varphi_{pu}^2 = .75$ (i.e., average probability of solution 3/4)
    - N.B.: $\sqrt{.75} = .87 \approx .85$ minimum for Tucker's φ
- Hence, the following partitioning …

Four Sectors

- First: Effectively blind
  - $.00 \leq \varphi_{pu}^2 \leq .25 = Q_1$ (1st quartile)
- Second: Mostly blind but partially sighted
  - $.25 < \varphi_{pu}^2 \leq .50 = Q_2$ (2nd quartile)
- Third: Mostly sighted but partially blind
  - $.50 < \varphi_{pu}^2 \leq .75 = Q_3$ (3rd quartile)
- Fourth: Effectively sighted
  - $.75 < \varphi_{pu}^2 \leq 1.0$
  - "pure" sighted if $\varphi_{pu}^2 > .90 \approx .95^2$

Connection with Typology

- φ_{pu}^2 *tends* to increase with more variant Types 1 and 2 (ps > 0 and us > 0)
- φ_{pu}^2 *always* decreases with more variant Types 3 and 4 (ps > 0 and us = 0)
- φ_{pu}^2 *always* decreases with more variant Types 5 and 6 (ps = 0 and us > 0)
- φ_{pu}^2 *neither* increases nor decreases with variant Types 7 and 8 (ps = 0 and us = 0)

Selection Procedures

- External versus Internal
  – Introduces no complications
- Simultaneous versus Sequential
  – Introduces complications

Sequential Selection

- Need to add a index for consecutive trials to allow for changes in the parameter values:
- $p_{1t}, p_{2t}, p_{3t}, \dots p_{it}, \dots p_{kt}$
- $u_{1t}, u_{2t}, u_{3t}, \dots u_{it}, \dots u_{kt}$
- $v_{1t}, v_{2t}, v_{3t}, \dots v_{it}, \dots v_{kt}$
- where $t = 1, 2, 3, \dots n$ (number of trials)
- Then still, $0 \leq \varphi_{pu}{}^2(t) \leq 1$, but
- $\varphi_{pu}{}^2(t) \rightarrow 1$ as $t \rightarrow n$ (Type 3 to Type 8)

Caveat: Pro-Sightedness Bias

- Because φ_{pu}^2 increases with Type 2 though $v_i = 0$, it could reflect chance concurrences between **p** and **u**
  - e.g., lucky response biases
- Hence, superior measure would use
  - $\varphi_{pw}^2 = (\sum p_i w_i) / (\sum p_i^2 \sum w_i^2)$,
  - where $w_i = u_i v_i$, and hence $\varphi_{pw}^2 < \varphi_{pu}^2$
- But v_i is seldom known, so …

Concrete Illustrations

- Edison’s “drag hunts”
- Picasso’s horse sketches for *Guernica*
- Kepler’s Third Law
- Watson’s discovery of the DNA base pairs

Edison's "drag hunts"

- For lamp filaments, battery electrodes, etc.
- Incandescent filament utility criteria:
  - (1) low-cost,
  - (2) high-resistance,
  - (3) brightly glow 13½ hours, and
  - (4) durable

Edison's "drag hunts"

- Tested hundreds of possibilities:
  - 100 trial filaments: $\varphi_{pu}^2 \approx .01$ (1^{st} percentile)
  - 10 trial filaments: $\varphi_{pu}^2 \approx .1$ (1^{st} decile)
- These two estimates do not require equiprobability, only *p*-*u* "decoupling"
- e.g., same results emerge when both **p** and **u** are vectors of random numbers with positively skewed distributions (i.e., the drag hunts are "purely blind")

Picasso's *Guernica* Sketches

- 21 horse sketches represent the following solution variants with respect to the head:
  - X_1 = head thrusting up almost vertically: 1, 2, and 3 (top)
  - X_2 = head on the left side, facing down: 4 and 20
  - X_3 = head facing up, to the right: 5, 6, 7, 8, 9, and 11
  - X_4 = head upside down, to right, facing down, turned left: 10, 12, and 13
  - X_5 = head upside down, to left, facing down, turned left: 15
  - X_6 = head upside down, to right, facing down, pointed right: 17
  - X_7 = head level, facing left: 3 (bottom), 18 (top), 18 (bottom), 28, and 29
- Yielding …

Probabilities and Utilities

- $p_1 = 3/21 = .143$
- $p_2 = 2/21 = .095$
- $p_3 = 6/21 = .286$
- $p_4 = 3/21 = .143$
- $p_5 = 1/21 = .048$
- $p_6 = 1/21 = .048$
- $p_7 = 5/21 = .238$

- $u_1 = 0$
- $u_2 = 0$
- $u_3 = 0$
- $u_4 = 0$
- $u_5 = 0$
- $u_6 = 0$
- $u_7 = 1$

Picasso's *Guernica* Sketches

- Hence, $\varphi_{pu}^2 \approx .293$ (2nd sector, lower end)
- If complications are introduced, e.g.,
  - differentiating more horse variants so $k > 7$,
  - assuming that there are separate whole-part utilities,
- then $\varphi_{pu}^2 < .293$ (viz. 1st sector)

(Re)discovering Kepler's 3rd Law
Systematic Search

- D^1/T^1 $u_1 = 0$
- D^1/T^2 $u_2 = 0$
- D^2/T^1 $u_3 = 0$
- D^2/T^2 $u_4 = 0$
- D^2/T^3 $u_5 = 0$
- D^3/T^2 $u_6 = 1$
- D^3/T^3 $u_7 = 0$

- $\varphi_{pu}^2(1) = .143$
- $\varphi_{pu}^2(2) = .167$
- $\varphi_{pu}^2(3) = .200$
- $\varphi_{pu}^2(4) = .250$
- $\varphi_{pu}^2(5) = .333$
- $\varphi_{pu}^2(6) = .500$
- Not tested

(Re)discovering Kepler's 3rd Law BACON's Heuristic Search

D^1/T^1	$u_1 = 0$	$\varphi_{pu}^2(1) = .143$
D^1/T^2	$u_2 = 0$	$\varphi_{pu}^2(2) = .167$
D^2/T^1	$u_3 = 0$	Not tested
D^2/T^2	$u_4 = 0$	Not tested
D^2/T^3	$u_5 = 0$	Not tested
D^3/T^2	$u_6 = 1$	$\varphi_{pu}^2(6) = .500$
D^3/T^3	$u_7 = 0$	Not tested

Watson's Discovery of the DNA Base Pairs

- Four bases (nucleotides):
  - two purines: adenine (A) and guanine (G)
  - two pyrimidines: cytocine (C) and thymine (T)
- Four variants:
  - X_1 = A-A, G-G, C-C, and T-T
  - X_2 = A-C and G-T
  - X_3 = A-G and C-T
  - X_4 = A-T and G-C

Watson's Discovery of the DNA Base Pairs

- $u_1 = 0$, $u_2 = 0$, $u_3 = 0$, and $u_4 = 1$
- where only the last explains Chargaff's ratios (i.e., %A/%T = 1 and %G/%C = 1)
- But according to Watson's (1968) report:
- at $t = 1$, $p_{11} >> p_{21} \approx p_{31} \approx p_{41}$: e.g.,
  - $p_{11} = .40$, $p_{21} = p_{31} = p_{41} = .20$, $\varphi_{pu}{}^2(1) = .143$
  - $p_{11} = .28$, $p_{21} = p_{31} = p_{41} = .24$, $\varphi_{pu}{}^2(1) = .229$

Conclusions

- First, creative solutions entail Type 2 variants with
  - (a) low generation probabilities (high novelty),
  - (b) high utilities (high usefulness), and
  - (c) low selection expectations (high surprise)

Conclusions

- Second, creative Type 2 variants can only be distinguished from noncreative Type 3 variants by implementing BVSR
- That is, because the creator does not know the utility in advance, Type 2 and Type 3 can only be discriminated via generation and test episodes

Conclusions

- Third, φ_{pu}^2 provides a conservative estimate of where solution variant sets fall on the blind-sighted continuum.
- When φ_{pu}^2 is applied to real problem-solving episodes, $\varphi_{pu}^2 \leq .5$
- Moreover, variant sets seldom attain even this degree of sightedness until BVSR removes one or more Type 3 variants